Contents

Preface vii

Abbreviations ix

Chapter 1. The Minimum You Need to Know About Stochastic Processes 1

1.1. Operations on random variables 1
1.2. Stochastic processes 4
1.2.1. Summary of second-order statistics of stochastic processes 6
1.2.2. Stationary (strict sense) stochastic processes 8
1.2.3. Mean-stationary stochastic processes 8
1.2.4. Wide-sense stationary (WSS) stochastic processes 9
1.2.5. Stochastic processes with independent, stationary increments 10
1.3. Specificities of second-order stochastic processes 11
1.3.1. Convergence in quadratic mean (qm) sense 11
1.3.2. Continuity (qm) for a stochastic process 12
1.3.3. Derivability (qm) for a stochastic process 12
1.3.4. Stochastic integration 13
1.4. White noise stochastic process model 15
1.4.1. White noise model (discrete-time) 15
1.4.2. White noise model (continuous time) 16
1.4.3. Example: uniform white noise 16

Chapter 2. Stochastic Discrete Domain Convolutive Integration Models 19

2.1. Motivation 19
2.2. Fractional-order stochastic 2D integral fields from convolutions 21

2.2.1. Discrete domain fractional-order spatio-stochastic integration 21
2.2.2. Fractionally integrated fields from doubly indexed Gegenbauer polynomials 29
2.3. Spatial Gegenbauer fields: proof of proposition 2.2 35

Chapter 3. Stochastic Continuous Domain Convolutive Integration Models 39

3.1. Continuous spatial domain fractional-order integration models 39
3.1.1. Generalized fractional Brownian fields (GFBF) 40
3.1.2. On convolution of MFBF (to GFBF) 43
3.2. On convolutions of modulated fractional Brownian motions 48
3.2.1. Context 48
3.2.2. PSD association with FBMM $\boldsymbol{X}_{\mathcal{H}_k,u_k}$ via that of $\mathbf{Y}_k$ 50
3.2.3. Convolution between FBMM $\boldsymbol{X}_{\mathcal{H}_1,u_1}, \boldsymbol{X}_{\mathcal{H}_2,u_2}$ via characterization of $\mathbf{Y}_1 \star \mathbf{Y}_2$ 53

Chapter 4. On Machine Learning of Very Long-Range Spatial Dependence Structures 55

4.1. PSD-based learning for multi-fractional interaction identification 57
4.1.1. Multi-fractional interaction identification problem 57
4.1.2. State-of-the-art: direct and Fourier-based transfer learning of multi-fractional interaction features 57
4.2. Multi-fractional interaction identification from PSD pole counting 59
4.2.1. GFBF PSD poles determine the identification problem 59
4.2.2. Stochastic interaction learning constrained to PSD feature extraction 61
4.3. Conclusion 65

References 67

Index 71